

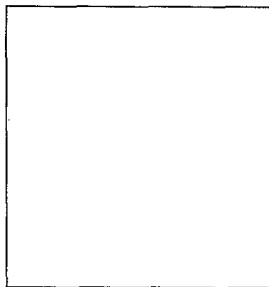
ELECTROWEAK VECTOR BOSON PRODUCTION IN JOINT RESUMMATION

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We study the application of the joint resummation to electroweak boson production at hadron colliders. The joint resummation formalism resums both threshold and transverse momentum corrections to the transverse momentum distribution at next-to-leading logarithmic accuracy. We obtain a good description of the transverse momentum distribution of Z bosons produced at the Tevatron collider.

1 Introduction

Electroweak boson production at hadron colliders serves as one of the major testing grounds for resummation techniques of perturbative QCD. In this talk we will describe the application of the recently proposed joint resummation formalism¹ to these type of processes and compare theoretical predictions of the formalism with the data on Z boson production at the Tevatron collider.

It is well-known that the partonic subprocesses calculated using perturbation theory acquire logarithmic corrections due to gluon emission. The corrections arise from cancellations between virtual and real contributions at each order in perturbation theory and become large if a distribution is inspected near the phase-space boundary. The two prominent examples are *threshold* and *recoil* corrections, encountered in the partonic hard-scattering cross sections. The threshold corrections of the form $\alpha_s^n \ln^{2n-1}(1-z)/(1-z)$ become large when the partonic c.m. energy approaches the invariant mass Q of the produced boson, $z = Q^2/\hat{s} \rightarrow 1$. The recoil corrections, in turn, are of the form $\alpha_s^n \ln^{2n-1}(Q^2/Q_T^2)$ and grow large if the transverse momentum carried by the produced boson is very small, $Q_T \ll Q$. Thus, sufficiently close to the phase-space boundary, i.e. in the limit of soft and/or collinear radiation, fixed-order perturbation theory is bound to fail. A proper treatment of higher-order corrections in this limit requires resummation of logarithmic corrections to all orders. Such techniques are well established both in the

threshold^{2,3} and in the recoil^{4,5,6,7} case.

However, resummation of recoil and threshold corrections separately can lead to opposite effects, i.e. suppression or enhancement of the partonic cross section, respectively. A full analysis of soft gluon effects in transverse momentum distributions $d\sigma/dQ^2 dQ_T^2$ should therefore take these two types of corrections simultaneously into account. A joint treatment of these corrections was proposed in^{1,8}. It relies on a novel refactorization of short-distance and long-distance physics at fixed transverse momentum and energy¹. Similarly to standard threshold and recoil resummation, exponentiation of logarithmic corrections occurs in the impact parameter b space, Fourier-conjugated to transverse momentum Q_T space, and Mellin- N moment space, conjugated to z space. This time both transforms are present, resulting in a final expression which obeys energy and transverse momentum conservation. Consequently, phenomenological evaluation of the joint resummation expressions requires providing prescriptions for inverse transforms from N and b spaces. This also involves specifying a border between resummed perturbation theory and the nonperturbative regime, by analyzing and parameterizing nonperturbative effects. Moreover, to fully define the expressions a procedure for matching between the fixed-order and the resummed result needs to be specified. In the following talk we will discuss these topics in more detail.

The jointly resummed cross section

In the framework of joint resummation¹ we derive the following expression at next-to-leading logarithmic accuracy for electroweak annihilation^{1,9}:

$$\begin{aligned} \frac{d\sigma_{AB}^{\text{res}}}{dQ^2 dQ_T^2} &= \sum_a \sigma_a^{(0)} \int_{C_N} \frac{dN}{2\pi i} \tau^{-N} \int \frac{d^2 b}{(2\pi)^2} e^{i\vec{Q}_T \cdot \vec{b}} \\ &\times C_{a/A}(Q, b, N, \mu, \mu_F) \exp \left[E_{a\bar{a}}^{\text{PT}}(N, b, Q, \mu) \right] C_{\bar{a}/B}(Q, b, N, \mu, \mu_F), \end{aligned} \quad (1)$$

where $\sigma_a^{(0)}$ denotes the Born cross section, $\tau = Q^2/S$, and Q is the invariant mass of the produced boson. The flavour-diagonal Sudakov exponent $E_{a\bar{a}}^{\text{PT}}$, at the next-to-leading logarithmic (NLL) accuracy in N and b was derived in⁹:

$$E_{a\bar{a}}^{\text{PT}}(N, b, Q, \mu, \mu_F) = - \int_{Q^2/\chi^2}^{Q^2} \frac{dk_T^2}{k_T^2} \left[A_a(\alpha_s(k_T)) \ln \left(\frac{Q^2}{k_T^2} \right) + B_a(\alpha_s(k_T)) \right]. \quad (2)$$

Dependence on the renormalization scale is implicit in Eq. (2) through the expansion of $\alpha_s(k_T)$ in powers of $\alpha_s(\mu)$. $E_{a\bar{a}}^{\text{PT}}$ has the classic form of the Sudakov exponent in the recoil-resummed Q_T distribution for electroweak annihilation, with the A and B functions defined as perturbative series in α_s ^{4,5,6,7}. The coefficients in the expansion of these functions are the same as in the pure Q_T resummation and are known from comparison with fixed-order calculations^{10,11,12}; at the NLL only the logarithmic terms with $A^{(1)}$, $B^{(1)}$ and $A^{(2)}$ coefficients contribute,

$$\begin{aligned} A_a^{(1)} &= C_F, & B_a^{(1)} &= -\frac{3}{2}C_F, \\ A_a^{(2)} &= \frac{C_F}{2} \left[C_A \left(\frac{67}{18} - \frac{\pi^2}{6} \right) - \frac{10}{9}T_R N_F \right]. \end{aligned} \quad (3)$$

The quantity $\chi(N, b)$ organizes the logarithms of N and b in joint resummation⁹

$$\chi(\bar{N}, \bar{b}) = \bar{b} + \frac{\bar{N}}{1 + \eta \bar{b}/\bar{N}}, \quad (4)$$

where η is a constant and we define

$$\bar{N} = N e^{\gamma_E}, \quad (5)$$

$$\bar{b} \equiv b Q e^{\gamma_E/2}. \quad (6)$$

with γ_E the Euler constant. As expected, with this choice of the form for $\chi(\bar{N}, \bar{b})$ the LL and NLL terms are correctly reproduced in the threshold limit, $N \rightarrow \infty$ (at fixed b), and in the recoil limit $b \rightarrow \infty$ (at fixed N).

The functions $\mathcal{C}(Q, b, N, \mu, \mu_F)$ in Eq. (1) are chosen to correspond to the jointly resummed cross section for large N and arbitrary b , and to Q_T resummation for $b \rightarrow \infty$, N fixed:

$$\mathcal{C}_{a/H}(Q, b, N, \mu, \mu_F) = \sum_{j,k} C_{a/j}(N, \alpha_s(\mu)) \mathcal{E}_{jk}(N, Q/\chi, \mu_F) f_{k/H}(N, \mu_F). \quad (7)$$

They are products of parton distribution functions $f_{k/H}$ at scale μ_F , an evolution matrix \mathcal{E}_{jk} and coefficients $C_{a/j}(N, \alpha_s)$ with a structure of a perturbative series in α_s . The first order expansions of the latter are given here by

$$C_{q/q}^{(1)}(N, \alpha_s) = 1 + \frac{\alpha_s}{4\pi} C_F \left(-8 + \pi^2 + \frac{2}{N(N+1)} \right) = C_{\bar{q}/\bar{q}}^{(1)}(N, \alpha_s), \quad (8)$$

$$C_{q/g}^{(1)}(N, \alpha_s) = \frac{\alpha_s}{2\pi} \frac{1}{(N+1)(N+2)} = C_{\bar{q}/g}^{(1)}(N, \alpha_s). \quad (9)$$

The matrix $\mathcal{E}(N, Q/\chi, \mu_F)$ represents the evolution of the parton densities from scale μ_F to scale Q/χ up to NLL accuracy⁹ in $\ln N$. It is derived from NLO solutions of standard evolution equations^{13,14}. By incorporating full evolution of parton densities (as opposed to only the leading N part of the anomalous dimension) the cross section (1) correctly includes the leading $\alpha_s^n \ln^{2n-1}(\bar{N})/N$ collinear non-soft terms to all orders. In fact, due to our treatment of evolution, expansion of the resummed cross section (1) in the limit $N \rightarrow \infty$, $b = 0$ returns all $\mathcal{O}(1/N)$ terms in agreement with the fixed-order result. Further comparison can be undertaken in the limit $b \rightarrow \infty$, $N = 0$ when our joint resummation turns into standard Q_T resummation. Consequently, the NLO transverse momentum distribution is recovered from the $\mathcal{O}(\alpha_s)$ expansion of the jointly resummed cross section exactly in the same way as in the Q_T resummation. Outside of these limits, a numerical comparison between the fixed-order and the expanded jointly resummed expression for $d\sigma/dQ_T$ at $\mathcal{O}(\alpha_s)$ shows, as expected, a very good agreement—especially in the small Q_T region.

Inverse transforms and matching

The jointly resummed cross section (1) requires defining inverse Mellin and Fourier transforms so that singularities associated with the Landau pole are avoided. Similarly to threshold resummation, there are also singularities in N space associated with parton distribution functions. A contour for the Mellin integral in (1) is chosen to be analogous to the 'minimal prescription' contour in threshold resummation¹⁵

$$N = C + ze^{\pm i\phi}. \quad (10)$$

For $\phi > \pi/2$, this results in an exponentially convergent integral over N in the inverse transform, Eq. (1) for all $\tau < 1$ ^{16,15}. The constant C has to be such that the contour lies to the right of the rightmost singularity of the parton distribution functions but left of the Landau pole.

The inverse Fourier integral from b space also suffers from the Landau singularity. We define this integral using the identity

$$\int d^2b e^{i\vec{Q}_T \cdot \vec{b}} f(b) = 2\pi \int_0^\infty db b J_0(bQ_T) f(b) = \pi \int_0^\infty db b [h_1(bQ_T, v) + h_2(bQ_T, v)] f(b), \quad (11)$$

and employing Cauchy's theorem to deform the integration contour along the real b axis into a contour in complex b plane^{8,9}. The auxiliary functions $h_{1,2}(z, v)$ are related to Hankel functions

and are defined by integrals in the complex θ -plane¹⁷:

$$\begin{aligned} h_1(z, v) &\equiv -\frac{1}{\pi} \int_{-iv\pi}^{-\pi+iv\pi} d\theta e^{-iz \sin \theta}, \\ h_2(z, v) &\equiv -\frac{1}{\pi} \int_{\pi+iv\pi}^{-iv\pi} d\theta e^{-iz \sin \theta}. \end{aligned} \quad (12)$$

The h functions distinguish between the positive and negative phases in Eq. (11). The b integral can thus be written as a sum of two contour integrals (plus contributions vanishing at large $|b|$, coming from closing the contours), one integral of the corresponding integrand with h_1 along a contour C_1 in the upper half of the b plane, the other integral of the integrand with h_2 along a contour C_2 in the lower half. The Landau pole can be avoided if one defines the contours in the following way:

$$C_1: \quad b = \begin{cases} t & (0 \leq t \leq b_c) \\ b_c - te^{-i\phi_b} & (0 \leq t \leq \infty) \end{cases} \quad C_2: \quad b = \begin{cases} t & (0 \leq t \leq b_c) \\ b_c - te^{i\phi_b} & (0 \leq t \leq \infty) \end{cases}, \quad (13)$$

where parameters b_c and ϕ_b are arbitrary as long as the contour does not run into the Landau pole singularity or singularities associated with the particular form (4) of the function χ .

In the joint resummation we adopt the following matching prescription between the resummed and the fixed-order result:

$$\frac{d\sigma}{dQ^2 dQ_T^2} = \frac{d\sigma^{\text{res}}}{dQ^2 dQ_T^2} - \frac{d\sigma^{\text{exp}(k)}}{dQ^2 dQ_T^2} + \frac{d\sigma^{\text{fixed}(k)}}{dQ^2 dQ_T^2}, \quad (14)$$

where $d\sigma^{\text{res}}/dQ^2 dQ_T^2$ is given in Eq. (1) and $d\sigma^{\text{exp}(k)}/dQ^2 dQ_T^2$ denotes the terms resulting from the expansion of the resummed expression in powers of $\alpha_s(\mu)$ up to the order k at which the fixed-order cross section $d\sigma^{\text{fixed}(k)}/dQ^2 dQ_T^2$ is taken. The above matching prescription in (N, b) space guarantees that no double counting of singular contributions occurs in the matched distribution.

Transverse momentum distribution for Z production

Joint resummation predictions for Z boson production compared with the latest CDF data from the Tevatron collider¹⁸ are shown in Fig. 1. Due to the contour integral prescription for performing inverse transforms, in the framework of joint resummation one does not require any extra nonperturbative information to obtain predictions. This is not the case in the standard Q_T resummation formalism, where nonperturbative parameters are introduced to make the theoretical expression well defined. As shown by the dashed line in Fig. 1 the joint resummation without any extra nonperturbative input already provides a good description of the data, except for the region of very small Q_T , where the nonperturbative effects are expected to play a significant role. However, the form of the nonperturbative input can be predicted within the joint resummation by taking the limit of small transverse momentum of soft radiation in the exponent, Eq. (2). Assuming moderate threshold effects the procedure gives a simple Gaussian parametrization $F_{NP}(b) = \exp(-gb^2)$. The value of the parameter $g = 0.8 \text{ GeV}^2$ is determined by fitting the predicted distribution to the data. It is very similar to the value obtained in Ref.¹⁹, where an extrapolation of the Q_T -resummed cross section to large b was made. The solid line in Fig. 1 represents predictions including the nonperturbative parametrization. In the large Q_T region, see Fig. 1b, the joint resummation formalism with the matching prescription (14) also returns a very good description of data without requiring an additional switching to pure fixed-order result, unlike in the standard Q_T resummation formalism.

In summary, we obtain a well-defined jointly resummed cross section, valid for all nonzero Q_T , which successfully describes the Q_T distribution of Z bosons produced at the Tevatron.

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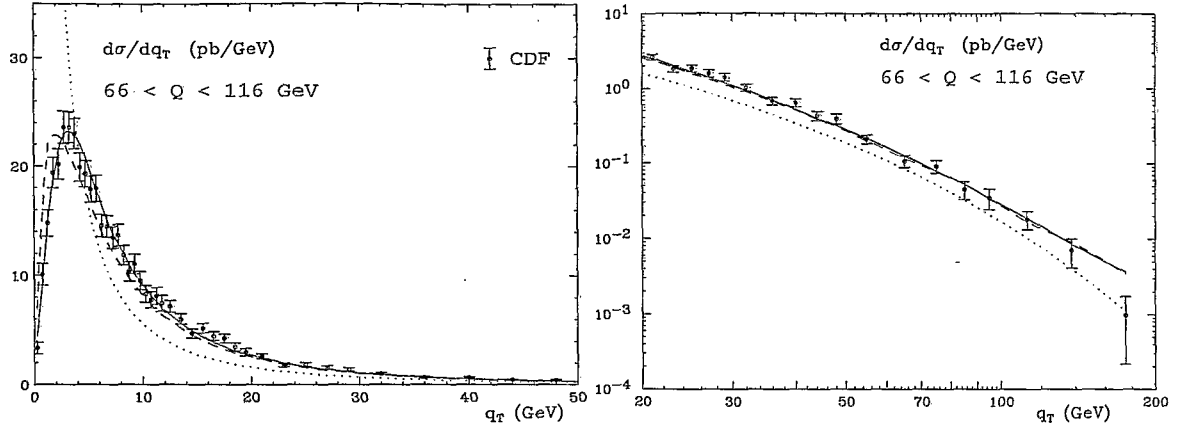


Figure 1: CDF data¹⁸ on Z production compared to joint resummation predictions (matched to the $\mathcal{O}(\alpha_s)$ result according to Eq. (14) without nonperturbative smearing (dashed) and with Gaussian smearing using the nonperturbative parameter $g = 0.8 \text{ GeV}^2$ (solid). The dotted line shows the fixed-order result. The normalizations of the curves (factor of 1.035) have been adjusted in order to give an optimal description. We use CTEQ5M parton distribution functions, $\mu = \mu_F = Q$ and $\phi = \phi_b = 25/32\pi$, $C = 1.3$, $b_c = 0.2/Q$.

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